Calculus Warm Up #7-4

1. Find the value(s) of c guaranteed by the mean value theorem for f on the given interval.

$f(x) = x^2 - 2x + 4$ on $[-1, 4]$

Check your answers: Ch. 4 review WS

1. Ab. Min @ (1, -5)
   Ab. Max @ (-1, -1) & (3, -1)

2. Ab. Min @ (5, -5)
   Ab. Max @ (2, 4)

3. critical #: - 4
   f is decreasing on $(-\infty, -4)$
   f is increasing on $(-4, \infty)$
   Min @ (- 4, - 6)

4. Min @ (0, 5)
   Max @ (2, 9)
   PI @ (1, 7)
   Concave up on $(-\infty, 1)$
   Concave down on $(1, \infty)$

5. a) 0 b) $\frac{1}{2}$ c) 1

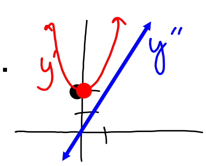
6. PI, Show f″ test to verify sign change which indicates a change in concavity.

7. Rel. Min @ (2, -12)
   verify with f′ or f″ test

8. $d = \frac{\sqrt{5}}{2}$

9. Verticals @ x = 0, x = 3, x = -3

   Horizontal @ $y = -\frac{2}{3}$

10. 

CH. 4 review Questions?

1. Locate the absolute extrema of $f(x) = x^2 - 2x - 4$ on [-1, 3]

2. Locate the absolute extrema of $f(x) = 4x - x^2$ on [0, 5]

3. Given $f(x) = x^2 + 8x + 10$, find the critical numbers of f, the open intervals o
decreasing, and locate all relative extrema.

4. List any extrema and/or points of inflection, and describe the concavity of $f(x) = 5 + 3x^2 - x^3$

5. Find the limits:

a) $\lim\limits_{x\to\infty} \dfrac{2x-6}{x^2-3x}$

b) $\lim\limits_{x\to\infty} \dfrac{\overbrace{(x^2+1)(x^2-1)}^{x^4-1}}{2x^4}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{x^4}{x^4} - \frac{1}{x^4}}{\frac{2x^4}{x^4}}$

$= \lim\limits_{x\to\infty} \dfrac{1 - \frac{1}{x^4}}{2}$

$= \dfrac{1-0}{2}$

$= \boxed{\dfrac{1}{2}}$

since: $\sqrt{x^2} = (\sqrt{x})^2$

let $\sqrt{x^2} = -x$

c) $\lim\limits_{x\to-\infty} \dfrac{-x}{\sqrt{x^2-2}}$

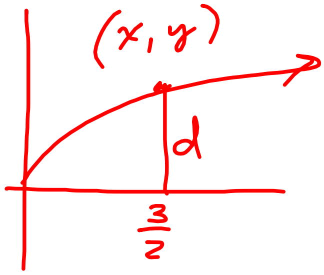
$\lim\limits_{x\to-\infty} \dfrac{\frac{-x}{-x}}{\sqrt{\frac{x^2}{x^2} - \frac{2}{x^2}}}$

6. Let $f(x) = (x-2)^5 - 15$. What type of point is $(2, -15)$?

7. Find a relative minimum for $f(x) = 2x^3 - 3x^2 - 12x + 8$. Verify that it is a minimum.

8. How close does the curve $y = \sqrt{x}$ come to the point $\left(\frac{3}{2}, 0\right)$?

minimize d.



$$d = \sqrt{\left(x - \tfrac{3}{2}\right)^2 + (y - 0)^2}$$

$$\rightarrow \sqrt{x^2 - 3x + \tfrac{9}{4} + (\sqrt{x} - 0)^2}$$

let $g(x) = x^2 - 2x + \frac{9}{4}$

$g'(x) = 2x - 2$

$0 = 2x - 2$

$x = 1$

$y = \sqrt{1}$

$y = 1$

$\boxed{(1, 1)}$

confirm min.

$g''(x) = 2 \quad +$ conc up everywhere

---

9) $\displaystyle \lim_{x \to \pm\infty} \frac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{9}{x^2}} + \frac{\frac{5}{x} - \frac{2x}{x}}{\frac{3x}{x}}$

$\displaystyle \lim_{x \to \pm\infty} \frac{\frac{1}{x} - \frac{1}{x^2}}{1 - \frac{9}{x^2}} + \frac{\frac{5}{x} - 2}{3}$

Last of Chapter 4, Today: 4.9

-Differentials

-Error propagation

-Differential formulas

We defined the derivative as:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \qquad \frac{dy}{dx} = f'(x)$$

There are applications where it will be useful to think of dy and dx separately:

Definition of differentials

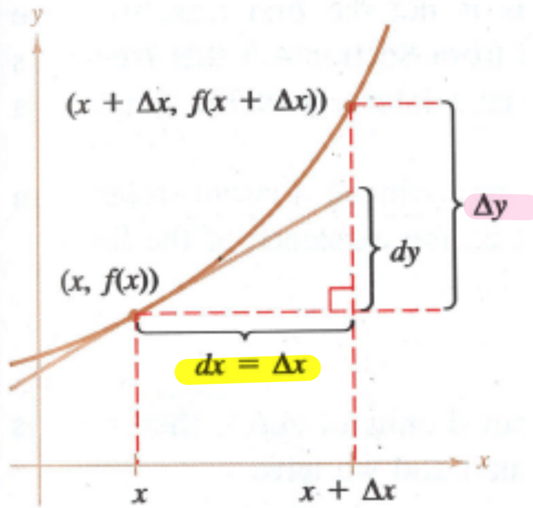
dy = the differential of y   dx = the differential of x

$$\boxed{dy = f'(x)\,dx}$$

Note that in this definition dx can have any nonzero value. In most applications of differentials, however, we choose dx to be small and we denote this choice by $dx = \Delta x$.

One use of differentials is to approximate the change in $f(x)$ that corresponds to a change in x as shown

$$\Delta y = f(x + \Delta x) - f(x).$$



EXAMPLE 1 Comparing Δy and dy

Let $y = x^2$. Find dy when $x = 1$ and $dx = 0.01$. Compare this value to Δy when $x = 1$ and $\Delta x = 0.01$.

$\frac{dy}{dx} = 2x$

$dy = 2x\,dx$

$dy = 2(1)(0.01)$

$dy = 0.02$

$\Delta y = f(x+\Delta x) - f(x)$

$\Delta y = f(1+0.01) - f(1)$

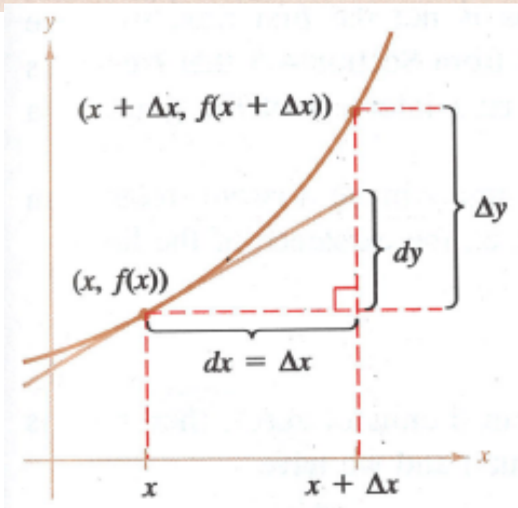
$\Delta y = (1.01)^2 - 1^2$

$\Delta y = 0.0201$

Let $y = f(x)$ be differentiable at x such that $f'(x) \neq 0$. If $dx = \Delta x$, then

$$\lim_{\Delta x \to 0} \frac{\Delta y}{dy} = 1$$

where $dy = f'(x)\,dx$ and $\Delta y = f(x + \Delta x) - f(x)$.



An application of differentials:

Error propagation

Physicists and engineers tend to make liberal use of the approximation of Δy by dy. One way this occurs in practice is in the estimation of errors propagated by physical measuring devices. For example, if we let x represent the measured value of a variable and let $x + \Delta x$ represent the exact value, then Δx is the *error in measurement*. Finally, if the measured value x is used to compute another value $f(x)$, then the difference between $f(x + \Delta x)$ and $f(x)$ is called the **propagated error.**

$$\underbrace{f(x + \overbrace{\Delta x}^{\text{measurement error}})}_{\text{exact value}} - \underbrace{f(x)}_{\text{measured value}} = \underbrace{\Delta y}_{\text{propagated error}}$$

EXAMPLE 2 Estimation of error p.215

The radius of a ball bearing is measured to be $\boxed{r = 0.7 \text{ inch}}$, as shown in Figure 4.57. If the measurement is correct to within 0.01 inch, estimate the propagated error in the volume V of the ball bearing.

$$V = \frac{4}{3}\pi r^3$$

Possible error in measurement

$$\Delta r = dr = \pm 0.01$$

$$\frac{dV}{dr} = 4\pi r^2$$

Estimate of Propagated Error:

$$\Delta V \approx dV$$

$$dV = 4\pi r^2 dr$$

$$dV \approx 4\pi(0.7)^2(\pm 0.01)$$

$$\approx \pm 0.06158 \text{ in}^3$$

Relative Error:

Compare dV to V $\longrightarrow$

$$\frac{dV}{V} = \frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3}$$

$$\frac{dV}{V} = \frac{3\,dr}{r}$$

$$\approx \frac{3}{0.7}(\pm 0.01)$$

$$\approx \pm 0.0429$$

$$\approx 4.29\%$$

EXAMPLE 3 Finding differentials

p. 216

Function	Derivative	Differential
(a) $y = x^2$	$\frac{dy}{dx} = 2x$	$dy = 2x\,dx$
(b) $y = 2x^3 + x$	$\frac{dy}{dx} = 6x^2 + 1$	$dy = (6x^2 + 1)\,dx$
(c) $y = \sqrt{2x+1}$	$\frac{dy}{dx} = \frac{1}{\sqrt{2x+1}}$	$dy = \left(\frac{1}{\sqrt{2x+1}}\right)dx$
(d) $y = \frac{1}{x}$	$\frac{dy}{dx} = -\frac{1}{x^2}$	$dy = -\frac{dx}{x^2}$

The notation in Example 3 is called the **Leibniz notation** for derivatives and differentials, named after the German mathematician Gottfried Wilhelm Leibniz (1646–1716). The beauty of this notation is that it provides us with an easy way to remember several important calculus formulas by making it seem as though the formulas were derived from algebraic manipulations of differentials. We will encounter several instances of this later in the text (in substitutions, inverse functions, parametric equations, and polar coordinates).

Differential of a composite function

$dy = f'(x)\,dx$

Use the Chain Rule to find dy for $y = (3x^2 + x)^4$.

$$dy = 4(3x^2+x)^3(6x+1)\,dx$$

Use the Chain Rule to find dy for $y = \sqrt{x^2 + 1}$.

$$dy = \frac{1}{2}(x^2+1)^{-1/2}(2x)\,dx$$

$$dy = \frac{x}{\sqrt{x^2+1}}\,dx$$

$\Delta y = f(x + \Delta x) - f(x)$ $\qquad$ $dy = f'(x)\,dx$

as $\Delta x \to 0$

$\Delta y \approx dy$

$$f(x + \Delta x) - f(x) \approx f'(x)\,dx$$

$+f(x)$ $\qquad$ $+f(x)$

$$f(x+\Delta x) \approx f(x) + f'(x)dx$$

Use differentials to estimate without a calculator:

$\sqrt{16.5}$

Let $f(x) = \sqrt{x}$ $\qquad$ Let $x = 16$ and $\Delta x = 0.5$

$f'(x) = \frac{1}{2\sqrt{x}}$

$$f(x + \Delta x) \approx f(x) + f'(x)\,dx$$

$$\sqrt{16.5} \approx f(16) + [f'(16)](0.5)$$

$$\sqrt{16.5} \approx 4 + \frac{1}{2(4)} \cdot \frac{1}{2}$$

$$\sqrt{16.5} \approx 4 + \frac{1}{16}$$

$$4.062 \approx 4.063$$

HW: p. 218 # 1 - 9 odd,
13, 19 -23 odd

Note: the worked-out solution for problem 21 is actually the solution for problem 20!!